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Study of Input Space for State Estimation of High-Rate Dynamics

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Abstract
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Keywords
High-rate, Microsecond, Millisecond, State Estimation, Observers, Fast Dynamics

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RESEARCH ARTICLE

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Study of input space for state estimation of high-rate dynamics

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Summary

High-rate dynamic systems are defined as systems being exposed to highly dynamic environments that comprise high-rate and high-amplitude events. Examples of such systems include civil structures exposed to blast, space shuttles prone to debris strikes, and aerial vehicles experiencing in-flight changes. The high-rate dynamic characteristics of these systems provides several possibilities for state estimators to improve performance, including a high potential to reduce injuries and save lives. In this paper, opportunities and challenges that are specific to state estimation of high-rate dynamic systems are presented and discussed. It is argued that a possible path to design of state estimators for high-rate dynamics is the utilization of adaptive data-based observers but that further research needs to be conducted to increase their convergence rate. An adaptive neuro-observer is designed to examine the particular challenges in selecting an appropriate input space in high-rate state estimation. It is found that the choice of inputs has a significant influence on the observer performance for high-rate dynamics when compared against a low-rate environment. Additionally, misrepresentation of a system dynamics through incorrect input spaces produces large errors in the estimation, which could potentially trick the decision-making process in a closed-loop system in making bad judgments.

KEYWORDS

fast dynamics, high-rate, microsecond, millisecond, observers, state estimation

1 | INTRODUCTION

High-rate dynamic systems are systems being exposed to highly dynamic environments, which comprise of high-rate and high-amplitude events. When exposed to these highly dynamic events, a system can undergo rapid changes. Examples of such systems include hypersonic vehicles and impact protection systems. The ability of an estimator to sense, analyze, and predict the state or health of high-rate dynamic systems could be invaluable to mitigate intolerable costs of human life or economic loss resulting from unintended failure.[1] If the system is capable of identifying and adapting to the change in a timely manner, control decisions, such as active mitigation strategies, can be undertaken to prevent further damage and complete failure.[2]

Advances in estimation and control theory, along with computer sciences, enable the development of observers with rapid convergence for estimation. Such observers have the potential to produce smarter, safer, and more effective systems capable of responding to real-time events. Although conventional estimators are not robust to noise and uncertainty, algorithms have been developed to handle such complexities although not without penalties.[3] In the presence of noise, a

Kalman filter (KF)-type observer can be used to obtain accurate estimates but with added computational costs. Likewise, methods have been developed to overcome heavy computation such as the uniform robust exact observers,[4] objective functions formulated from modal data,[5] and self-tuning fusion KFs.[6] Through these efforts, it is undeniable that modern research in state estimation has been geared towards producing faster and more efficient observers for systems with various levels of complexity.

With the growing number of applications needing estimators on high-rate dynamic systems, the research area of state estimation will soon require algorithms that can robustly estimate the states of high-rate dynamic systems. The objective of this paper is to illustrate the importance of addressing the high-rate dynamic-specific challenges, with a particular attention on the importance that the input space of an observer may play in high-rate dynamics state estimation.

Important challenges in high-rate dynamic systems will be introduced. A path to high-rate state estimation is discussed, which leverages adaptive observers (AOs) as a promising solution for complex nonstationary systems. The influence of the input space on a high-rate state estimator will be studied, and it will be shown that its proper design can substantially enhance the performance of AO. This investigation will be conducted through simulations of a neuro-observer on high-rate laboratory experimental data.

2 | APPLICATIONS FOR STATE ESTIMATION OF HIGH-RATE DYNAMICS

As scientists and engineers are continually developing faster and more powerful mechanical and civil systems, the concerns for safety is also growing. High-rate systems operate at speeds faster than human reflexes. Thus, in order to operate safely, they require the incorporation of smarter systems capable of making decisions that will protect the lives of the operators and/or surrounding personnel, as well as the financial investments in these high-rate systems. This section discusses engineering systems for which state estimation of high-rate dynamics can be particularly useful. Examples include civil structures exposed to blast, space shuttle debris prone to debris strikes, and aerial vehicle experiencing in-flight changes. They are organized into two categories: (a) impact detection and mitigation and (b) in-flight monitoring and rapid guidance adaptability.

2.1 | Impact detection and mitigation

Designing structures to withstand blast is a growing area of research due to the increased number of terrorist attacks. Military installations are not the only targets as seen in the historical bombings on civilian buildings such as the World Trade Center in 1993 and the Alfred P. Murrah Federal Building in 1995. Sources of blast loads for civil structures are not limited to terrorist attacks but also include accidental explosions caused by gas leaks, vehicular accidents, and chemicals mishaps. Blast creates a shock wave of energy that often exceeds the yield strength of the structural material, causing damage and jeopardizing structural integrity, and may result in partial or total collapses. Blast mitigation is typically conducted by passive strategies, including friction elements, laminated windows, hardened concrete, and more.[7–9] Semi-active and active control methods are also being investigated.

An example of an active blast mitigation method can be found in Wadley et al.[10] The authors proposed a pre-compressed cellular material that deploys before the blast wave arrival. The blast is detected by electromagnetic emission sensors milliseconds before arrival. The compressed material is then deployed by high-speed actuators. The compressibility of the material absorbs the shock waves, and the deployment of the material causes momentum cancellation. This active method is preferred over passive methods because it allows blast mitigation using far less materials. Although simulations showed promise, it was acknowledged that the release of the cellular material just prior to the arrival of the blast impulse would significantly improve the performance of the proposed procedure. The engineering of such reactive system would require the development of state estimators of high-rate dynamics. Specifically, the arrival time of a blast produced by a 10-kg TNT varies from approximately 0.3 ms at a 1 m range to approximately 100 ms at a 40 m range. It follows that the reactive system would necessitate sensing, estimation, and actuation below this 0.3–100 ms range.

Another example of technology for impact detection and mitigation is airbag systems. According to the United States Census Bureau, in 2009, 10.8 million motor vehicle accidents occurred of which 30,797 where fatal.[11] Not all accidents lead to serious injuries, but of the ones that do, airbags have played an important role in saving many lives.[12] The airbag control unit uses a combination of sensors such as accelerometers, impact sensors, wheel tachometers, and brake pressure sensors to make a quick decision on whether to deploy the airbags or not. The sensor signals enter an amplifier and are filtered before reaching the airbag control unit. Typical modern systems use threshold levels for each sensor. If the threshold is met for multiple sensors, the airbags trigger. However, at times, airbag systems are known to cause unnecessary or even fatal injuries.[13]

Improving the airbag system is an ongoing research. For instance, research in Lee et al.[14] on stereovision-based sensing methods uses stereo cameras combined with an intelligent algorithm to determine the occupant classification (small child or unsafe position) before the deployment of airbags. To accommodate for the large distortions in the data, a thin plate spline algorithm is used to calculate a smooth function and interpolate a surface. The child detection is conducted at the moment the vehicle is started to save on computation time. On the other hand, the unsafe position has to be identified during the impact while determining whether to disable or trigger the airbags in time. This system boasts a processing time of 960 ms. Another area that is being researched is in adaptive airbag deployment. This technology focuses on deployment force, deployment geometry, and stiffness of the airbag using multistage inflators and venting systems.[15] Fatal accidents occur at a very fast rate, forcing the human body to collide with the interior of the vehicle in a fraction of a second.[16] Like systems which may experience impact conditions could benefit from state estimators capable of estimating high-rate dynamics. In the case of airbag systems, estimation of the high-rate dynamics or impact location of a human being could result in rapid mitigation decisions to minimize or eliminate damage and losses by appropriate airbag deployment.

2.2 | In-flight monitoring and rapid guidance adaptability

Debris strike during a space shuttle launch can be catastrophic. The loss of Columbia in 2003, killing all crew members, arose from the impact of foam insulation to the leading edge of the left wing during the shuttle launch that caused a breach in the thermal protection system. The foam insulation separated 81.7 s into the flight at an altitude of 66,000 ft and traveling at a velocity of 705 m/s.[17] During reentry, heat pierced the leading edge insulation, which degraded the structure of the left wing. This resulted in a weakening of the structure causing loss of control and eventually destruction of the orbiter.[18] If the damage had been detected the instant it occurred, the launch could have been aborted, avoiding the catastrophic failure. Since then, National Aeronautics and Space Administration (NASA) has developed and uses a NASA debris radar system to target and track debris during ascent. The NASA debris system automatically detects and characterizes debris. It is capable of assigning a ballistic number to assess the material type, size, release location, and threat associated with the object.[19] Although this system is very sophisticated, it uses an off-site ground radar system. Ground intervention is not always possible due to communication delays or visibility issues, and knowledge of failures is required onboard for operational capabilities.[20] Because of the rate at which space shuttles travel during launch and reentry, even soft materials such as insulating foam behave differently and can pierce through metal panels, which may be very difficult to comprehend. Nonetheless, these phenomenons are real, and estimators to observe systems under high-rate events are necessary.

More generally, during a flight, aircrafts can experience system failures or damage arising from the failure of components or from an impact with a foreign object such as a bird strike, hail impact, and lightning strike. Such events can cause larger issues if it leads to uncertainty in the navigational decision-making capabilities. In the event of an in-flight failure or damage, rapid estimations using uncertain data are required to regain and maintain control of the aircraft. The faster the estimator, the less the error it needs to compensate for. There has been significant research towards fault detection, isolation and recovery and prognostics health management subsystems.[21] The traditional approach to providing fault estimation for aircraft has been to duplicate the hardware such as sensors, actuators, and flight control computers. An alternative to this redundancy is to use a model-based fault detection and diagnosis and generate redundant estimates of measured signals. To achieve robustness in the model-based methods, different techniques have been studied to include optimization methods, unknown input observers, sliding mode observers (SMOs), and geometric design approaches.[22]

Additionally, aerospace technology is moving towards the development of hypersonic passenger and military airframes, which will most likely face similar challenges of foreign object impacts. A typical Boeing passenger aircraft can operate at about 600 mph or equivalently about Mach 0.8. By definition, hypersonic refers to speeds of Mach 5 or greater.[23] High-rate estimators would have the benefit of increasing reaction time to anomalies by allowing faster decisions and decreasing risks associated with system failure. For example, Mach 5 at an altitude 10,000 m and −50 °C corresponds to an approximate speed of 1,500 m/s. If sampling at 1 Hz, there would be 1,500 m travel distance between data points. Alternatively, to obtain a 1 mm resolution while traveling, a sampling rate of 1.5 MHz would be necessary. At such a rate, for each sample an observer requires to converge, the vehicle travels 1 mm. And after the convergence of the observer, there will be time delays for decision-making computation and actuator reaction times.

3 | CHALLENGES ASSOCIATED WITH STATE ESTIMATION OF HIGH-RATE DYNAMICS

Estimating the dynamics states of complex structures with high-rate dynamics is a non-trivial task. By definition, high-rate dynamic systems require rapid state estimation to optimize guidance and save lives. Moreover, fast estimators should not be confused with state estimation of "fast" dynamics. For example, state observers for induction motors capable of convergence in the microsecond range have been reported in literature using an adaptive sliding observer and an extended Kalman filter (EKF) with computation times of 19 and 86 μs, respectively, for one update using a 250 MHz processor,[24] and using a Luenberger observer (LO), SMO, and EKF with computation times of 5, 5, and 100 μs, respectively, for one update using a 150 MHz processor.[25] Although this demonstrates that microsecond state estimation is possible and currently exists for some applications, the induction motor itself is not a high-rate dynamic system but instead a "fast" dynamic system. There are three key factors that differentiate high-rate dynamic systems from fast dynamic systems. High-rate dynamic systems may have

- Large uncertainties on the external loads. High-rate events will occur at an undetermined time with an uncertain amplitude. For example, a man-made blast cannot be predicted in time, and its amplitude largely depends on the explosive used and the detonation distance.
- High levels of nonstationarity and heavy disturbance. A high-rate load may provoke large changes in the system's states and could also significantly alter the system's dynamic parameters. For example, an unmanned aerial vehicle could lose a wing following a debris strike.
- Generation of unmodeled dynamics (noise) from change in mechanical configuration. The exposure to highly dynamic environments can cause significant changes, which can appear as noise in measurements. For example, noise may arise from cable movement, flexing of the electronics, and threaded interfaces, which may rattle from loss of torque under high-amplitude dynamics.

These factors seen in the form of system complexities (e.g., uncertain systems, noisy systems, and so forth) have been considered in many research, and solutions to specific complexities have been proposed.[5,6,26-30] Nevertheless, research is yet to address situations when all of these factors are combined, also known as the high-rate dynamics problem. A brief survey on observers is conducted in the next subsections to develop building blocks for a potential solution. The survey is intended to provide the reader with a general background and should be used to understand the possible impact that the choice of an observer may have on systems experiencing high-rate dynamics. It is divided between fixed observer and AO.

3.1 | Fixed observers and high-rate dynamic systems

The LO[31] is a classic observer used for linear systems with well-defined numerical models. The LO has proven itself to be valuable in the areas of system monitoring and regulations, detecting, and identifying failures in dynamic systems[32]. However, because they are heavily dependent on the mathematical model of the system, disturbances, dynamic uncertainties, and nonlinearities can be particularly challenging. The KF[33] can be used in linear applications where noise is present and characterized as Gaussian.[24] Although the KF typically produces estimations with higher accuracy than the LO, its implementation is more complex[25] due to possibly unmodeled nonlinearities in the system model and ill-conditioning of the covariance matrix.[34] The SMO[35] can be an alternative to provide enhanced robustness,[36-40] but it is sensitive to the choice of gain.[41] It also exhibits ripples in the presence of external noise.[25]

Variations of these typical fixed observers have been developed to broaden their applicability to more complex systems. For instance, EKF uses a linear approximation of the nonlinear system[42] by either taking the derivative of the nonlinear function or applying a Taylor series expansion to the desired order of the approximation. A challenge with the EKF is the linearization of the system making the corresponding propagation equations available only to the neighborhood of the estimate.[43] It requires longer time for convergence compared with the LO and SMO.[25] The EKF is also difficult to apply to nonlinear systems with time-varying parameters, particularly in the presence of noise.[26] Charles et al.[44] introduces the Utkin and Walcottzak variations of the SMO observers. Although robust, both of these variations struggle in the estimation of states when some inputs are unavailable. The high-gain observer (HGO) uses a sufficiently high observer gain that will guarantee good performance of the observer in terms of accuracy and speed of convergence.[45] For most cases, the HGO is used as an LO-type estimator with large gain, and the application is for estimating slowly varying states or inputs.[27] A disadvantage of the HGO is strong chattering when the gain is very large.[46] Lastly, the robust state estimator guarantees robustness of the designed estimator if time invariant nominal system matrices, constant filter design parameters, and stationary external inputs conditions are met. However, the computational complexity is comparable with that of the KFs.[47] Table 1 summarizes the observers discussed in this section in terms potential weaknesses for high-rate systems.

3.2 | AOs and high-rate dynamic systems

AOs are often modifications of the fixed observers discussed previously. For example, adaptive versions of the HGO and the EKF have been studied in Boizot and Busvelle.[48] The high-gain aspect of the HGO serves to eliminate the nonlinear part of the system while allowing rapid convergence, thus making it a good candidate for nonlinear systems. The EKF is useful to filter out system or process noise, therefore advantageous for field applications. The two observers were combined to create a high-gain extended Kalman filter (HG-EKF), which possesses the advantages of both estimators. The authors showed that the resulting HG-EKF could converge at a desired speed with the tuning of only one parameter while also being capable of noise rejection. Although promising, a key issue with the HG-EKF is with the exponential convergence occurring only at the beginning of the estimation process from the initial condition. Large perturbations in the system are difficult to estimate if they are not already modeled within the system. This issue was overcome with the development of the adaptive gain extended Kalman filter. The main advantages of this observer are the noise rejection and the ability to estimate perturbations. The trade-off is that the convergence rate of the adaptive gain extended Kalman filter is not as good as the HGO or HG-EKF due to the algorithm's complexity.

A coupled technique of system identification and state estimation has been proposed before. Plett[49] proposed a joint EKF method, where a weight EKF estimates the system parameters and a state EKF estimates the system states. Hu et al.[50]proposed an update to the joint EKF method to provide multiscale estimation. The two EFKs run concurrently to maintain accurate estimates of the state of charge and capacity of lithium-ion batteries, which degrade with age. In the multiscale case, a macro timescale is used to adapt the slowly varying parameters, whereas a micro timescale is used to adapt the fast varying states. By using a multiscale framework, the estimation process is accelerated. Nevertheless, from the high-rate view point, the lithium-ion battery is a slow time-varying system with smaller uncertainties. It is argued that the minimal representation by use of input space will indeed decrease computational times.

3.3 | A path to state estimation of high-rate dynamics

Further developments and extensions of state-of-the-art fixed observer and AO need to be conducted in order to broaden the applicability of observers to the problem of state estimation of high-rate dynamic systems. Although many paths could be undertaken to achieve that goal, the authors are noting the particular promise of adaptive data-based observers. AOs have been proposed to estimate the unmeasurable states for different classes of nonlinear systems.[51] They are typically characterized by asymptotic stability[52] but are known to have slower convergence rates. In related studies, Shahrokhi and Morari[52] attributed this problem to the utilization of single observation errors, whereas Khayati and Zhu[51] explained this slow convergence by the complexity of the adaptive law. AOs can be used to estimate states and parameters using

TABLE 1 Summary of typical fixed observers in terms of general applicability to the problem of state estimation of high-rate dynamics

Observer type	Performance summary	Applicable to high-rate dynamic systems?	Ref.
Luenberger observer	Very fast convergence rates for linear systems with precise system models	Not applicable, high-rate dynamic systems are nonlinear and uncertain	[32]
Sliding mode observer	High robustness and improved results for inaccurate models but sensitive to choice of gain limiting the convergence rate	Not suitable for high-rate dynamic systems	[36-41]
Extended Kalman filter	High accuracy for nonlinear systems with added noise but complex implementation leading to poor convergence rates	Poses challenges for real-time and high-rate applications	[25,34]
High-gain observer	Accurate and fast convergence rates for estimating slowly varying states or inputs	Not suitable for high-rate systems	[27,45]
Robust state estimator	Guarantees robustness for time invariant systems, constant filter design parameters, and stationary external inputs	Only applicable to stationary high-rate dynamic systems	[47]

input–output measurements, ideal for handling uncertainty in state estimation.[53] Methods include fuzzy logic[54] and neural network estimators.[55] The performance of data-based method is linked to the quality of data mining and interpretation algorithms, and an additional limitation can be found in the computational time required to achieve an appropriate estimate.[56]

The adaptable characteristics of a state estimator is deemed critical for high-rate dynamic systems given the high levels of uncertainties, nonstationarity, disturbance, and noise that they can undergo. Data-based solutions can also be particularly helpful at providing important flexibility by enabling state estimation without the reliance on a model, in particular for highly uncertain and nonstationary systems. For dynamic parameters undergoing large variations, a hybrid form between data- and model-based solutions could be used to accelerate convergence. For instance, a data-based technique would be used as a system identifier fed into a model-based technique. It follows that, although the authors selected an adaptive data-based approach as a possible path to the high-rate state estimation challenge, other viable techniques exist. Examples include dual methods of system identification and state estimation using KFs for time-varying systems,[50] modal-based methods including numerical algorithms for subspace state space system identification[57] and eigensystem realization algorithm,[58] and statistical methods including particle filters[59] and unscented KFs.[60]

The data-based adaptive state estimators are in essence black-box models that could be pre-trained or adapted sequentially. In the high-rate dynamics problem, pre-training could be difficult for a certain set of systems given the uncertainty on external loads. Sequential adaptation is also challenging, because the system is required to learn and perform on-the-spot. Algorithms capable of sequential adaptation and immediate performance have been studied in the field of structural control for mitigation of unknown excitations in uncertain systems.[61,62] However, sequential adaptation will typically yield larger initial errors and a slower convergence rate, and it follows that such observer needs to be designed appropriately.

The sequential adaptive learning problem can be simplified as the construction of a function characterizing an input–output system. This problem has been researched and addressed in many fields through various machine learning methods. When looking at high-rate dynamic systems as input–output systems, a key feature that distinguishes them is the vast quantity of input data. Even in the case of a single sensor, the very large sampling rate will result in the accumulation of a vast time series vector. It results that one needs to decide which part of this time series would be fed as input, or how many delayed observations will constitute the inputs such as

$$\begin{aligned}\hat{y}_k &= f(y_i(t), y_i(t-\tau), y_i(t-2\tau), \cdots, y_i(t-(d-1)\tau)) \\ &= f(v(\tau, d)),\end{aligned} \tag{1}$$

where $\hat{y}_k$ is the estimated state, y_i is an observation i, τ is a time delay, and d is the dimension of the input space v, with v termed the delay vector.[63] These parameters must be selected carefully in order to guarantee a certain level of performance. The procedure to select an input space in data-based techniques is often overlooked. The selection of input may influence computation time, adaptation speed, effects of the curse of dimensionality, understanding of the representation, and model complexity.[64–67] Available selection techniques include the filter methods, where the input selection is independent of the black-box model,[68] the wrapper methods, where the results from the black-box model are used to rank and select the inputs,[69] and the embedding methods, where selected inputs are used for adapting the representation.[70] Automatic input selection methods have been discussed,[67,71–74] but they are traditionally applied offline and necessitates pre-training. In the case of high-rate dynamic systems, as it will be demonstrated in the next section, the selection of the input space is critical to the observer's performance, and the optimal input space varies as a function of events.

4 | STUDY OF SYSTEM INPUT SPACE

We demonstrate the importance of the input space selection for high-rate dynamic systems by designing a neuro-observer and evaluating its performance as a function of different inputs. The design of the neuro-observer is presented in detail below. The subsequent subsection presents and discusses results from the simulation of the state estimation of a system experiencing high-rate dynamics. Note that the design of the neuro-observer was kept simple in order to focus the discussion on the importance of the input space selection. One could certainly design an adaptive observer that would provide, overall, better performance.

4.1 | Neuro-observer architecture

The neuro-observer is a single-layer wavelet neural network written

$$\hat{y}_k = \sum_{j=1}^{h} \gamma_j \phi_j(\boldsymbol{\nu}), \tag{2}$$

where h represents the number of nodes, γ the nodal weights of node j, and ϕ is the activation function taken as a Mexican hat wavelet:

$$\phi(\boldsymbol{\nu}) = \left(1 - \frac{\|\boldsymbol{\nu} - \boldsymbol{\mu}\|_2}{\boldsymbol{\sigma}^2}\right) e^{-\frac{\|\nu-\mu\|_2}{\sigma^2}}, \tag{3}$$

where μ and $\boldsymbol{\sigma}$ are the wavelet centers and bandwidths, respectively, and $\|\cdot\|_2$ is the 2-norm. The neuro-observer is designed to be capable of sequential adaptive learning, whereas no prior training is necessary. To do so, a self-organizing mapping architecture is adopted to minimize the network size.[75] This self-organization is conducted by adding a node if a new observation falls outside a Euclidean distance threshold D to the closest node. When a new node j is added, it is given a weight γ_j initially equal to zero, a center μ_j at the location of the new observation, and bandwidth $\boldsymbol{\sigma}_j$ of the newly added wavelet initially set at 10,000. The network is then put in an adaptation mode, where weights and bandwidths are adapted following a back-propagation rule. With the back-propagation rule, an adaptive parameter ζ is varied using

$$\dot{\zeta} = \Gamma_\zeta \frac{\partial(\boldsymbol{\gamma}^T \boldsymbol{\phi})}{\partial \zeta} \tilde{y}, \tag{4}$$

where $\tilde{y}$ is the observation error between the estimated and the measured state and Γ_ζ is the learning rate associated with the adaptive parameter ζ. The stability of the adaptation rule has been derived in Laflamme et al.[62] In a discrete notation, Equation 4 can be specialized for μ_j and $\boldsymbol{\sigma}_j$ at node j:

$$\begin{aligned} \gamma_j(k+1) &= \gamma_j(k) - \Gamma_{\gamma_j} \phi_j(\boldsymbol{\nu}) \tilde{y} \\ \sigma_j(k+1) &= \sigma_j(k) - \Gamma_{\sigma_j} \gamma_j \left(\frac{1}{\sigma_j^5} \mathrm{e}^{\frac{-\|\nu-\mu_j\|^2}{\sigma_j^2}} \left(4\sigma_j^2 \|\boldsymbol{\nu} - \mu_j\|^2 - 2\|\boldsymbol{\nu} - \mu_j\|^4\right) \right) \tilde{y}, \end{aligned} \tag{5}$$

where k is a discrete time step.

4.2 | Simulations

Simulations of the estimation using the neuro-observer were conducted on experimental data collected from an impact test series conducted on an electronic components package to demonstrate the importance of proper input space selection. The experimental setup is illustrated in Figure 1. On the right is the electronics system of interest. The systems

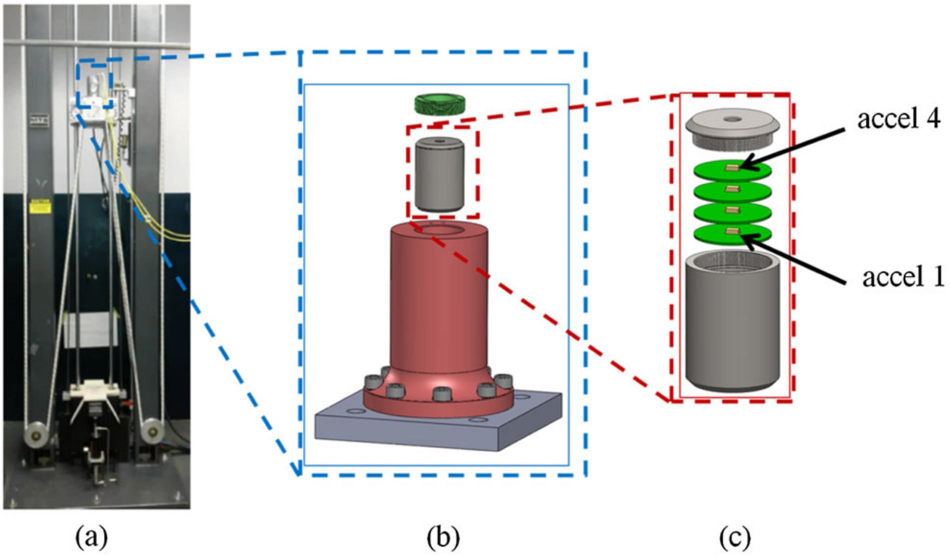

FIGURE 1 Experimental setup: (a) MTS-66 drop tower, (b) unit fixture, and (c) electronics unit

contains four circuit boards, each of the four boards have a surface mounted high-g shock accelerometer (Meggitt 72). These accelerometers are capable of accurately measuring acceleration upwards of 120,000 g_n or 120 kg_n.[76] The circuit boards are housed in a metal canister and filled with an electronics potting material. The system is secured in a metal fixture, shown in the middle of the figure, using a lock ring. On the left is the MTS-66 drop tower designed to generate a prescribed impact condition. Bungee cords are used to accelerate the table of the drop tower. The fixture housing the system is mounted on the drop tower table using bolts. The picture shows the table in the raised state. When the table brakes are released, the table along with the system accelerates downward and impacts the black mass at the bottom of the drop tower creating a mechanical shock. In this section, the acceleration data are presented in terms of g_n (1 g_n = 9.81 m/s^2 = 32.2 ft/s^2).

This problem contains many complexities making model-based techniques difficult to apply. The accelerometers are hardwired to the data acquisition system. When the table accelerates, so does the sensor cables, creating a violent whipping motion. Even though careful selection of cables was made, some noise is added from whipping of the cables. Additionally, whenever metal parts are coupled together in impact scenarios, such as in this case, noise is produced from the chattering of the parts. Uncertainties associated with this problem include unknown material high-rate response of the circuit boards and potting material, uncertainties in the exact placement of the accelerometers and circuit board spacing, and boundary conditions such as whether or not the potting material moves in relation to the housing. The exact input to the system resulting from the table impact is unknown. Only the measurement from the response of the accelerometers can be taken as certain. Note that such tests are considered as small impact tests within the shock dynamics community, and not enough energy was present to excite the sensor resonance. However, it is not uncommon in larger impact tests to notice disturbances created from sensor and/or system resonance.

The studied system bears many characteristics of a high-rate system. The way data is used in the simulation greatly simplifies the problem significantly by attempting to identify a representation estimating Accel 4 using Accel 1, where the inputs and outputs of the systems are known. Typical field applications would either have the outputs be unknown, for instance the estimation of velocity for acceleration data, or use the inputs to conduct system identification, for instance the identification of a stiffness value. Such realistic applications require the integration of additional steps and mathematics in the algorithm, which would drive the attention away from our discussion on the opportunities and limitations in the input space.

The impact is controlled by specifying a drop height and mitigating material. Event 1 is created with a drop height of 20 in. and 1/16 in thickness felt. The raw data were collected using a National Instruments PXI-6133 High-Speed cards with a sampling rate of 1 MHz coupled with a Precision Filters signal conditioning system 28144A quad-channel wideband transducer conditioner with an anti-aliasing filter of 204.6 kHz. Figure 2 shows three back-to-back tests of Event 1 that exhibit gradual increases in the response pointing to time-varying parameters possibly from the weakening of the glue that holds the accelerometers on to the circuit boards and/or from the electronics potting material de-bonding from the internal walls of the unit's metal canister. This confirms the high-rate dynamic nature of our system.

Figure 3 shows the experimental data from Accelerometers 1 (Accel 1) and 4 (Accel 4) of test 2, which the simulations of the estimation using the neuro-observer are conducted on. These accelerometers are selected because of the notable

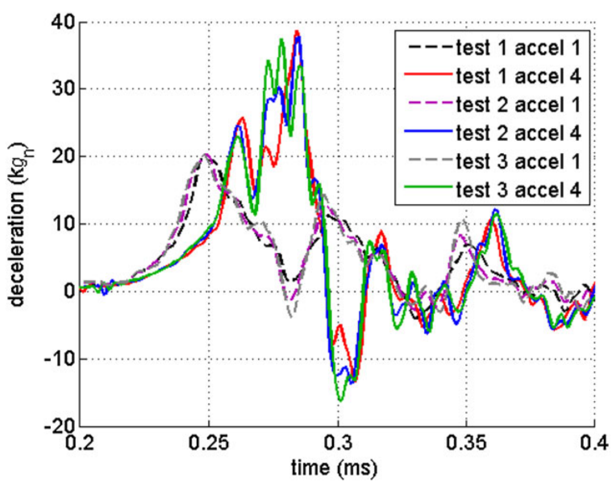

FIGURE 2 Response of three subsequent tests

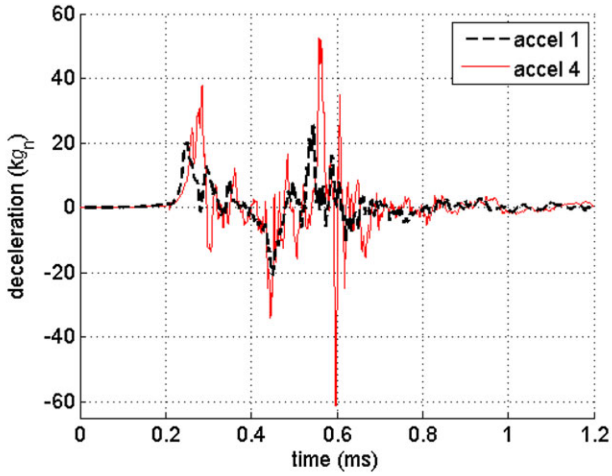

FIGURE 3 Experimental data from the accelerometer measurements

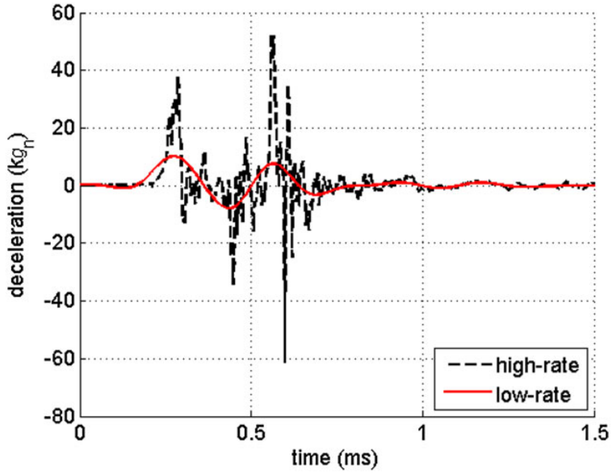

FIGURE 4 High- and low-rate data the simulations were conducted on

difference between the two responses. The impact occurs over 0.1 ms starting at about 0.2 ms, which in turn excites the system that responds over the next 0.6 ms. In the course of the dynamic event, an acceleration level greater than 60 kg_n is observed.

In the simulations, Accel 1 was used as the input, and Accel 4 was used as the output. The input was used to estimate the output using the neuro-observer. For the investigation, a lower rate event was synthetically created by filtering the experiment data at 5 kHz using a 4-pole Butterworth low pass filter, with the purpose to demonstrate the added importance of the input space for high-rate systems. From here on, high-rate data will refer to the raw data, and low-rate data will refer to the filtered data. Both time series are plotted in Figure 4.

Two different metrics were used to quantify the performance of the input space. The first performance metric J_1 of the estimator is taken as

$$J_1 = \frac{||(\hat{\boldsymbol{y}} - \ddot{\boldsymbol{x}}_4)||_2}{||\ddot{\boldsymbol{x}}_4||_2}, \tag{6}$$

with $\hat{\boldsymbol{y}} = \hat{\ddot{\boldsymbol{x}}}_4$ being the estimation of the measurement of Accel 4, $\ddot{\boldsymbol{x}}_4$. The $|| \cdot ||_2$ represents the 2-norm. By representing the error of the estimate as described in Equation 6, the error is normalized to fall within the values of [0, 1]. Additionally, normalizing the error helps to compare the errors between various simulations.

The second performance metric J_2 of the estimator is in terms of the convergence rate. The convergence rate is defined as the time it takes from the start of the impact (>100 g_n) to when the estimation error falls and remains within an error threshold. The error threshold was determined to be 5% and is governed by the variations in the data created by the experimental setup. Because the objective of the paper is to demonstrate the importance of the input space and not the optimality of the observer solution, the computational time is used as a performance metric. For the same reason, a study on the influence of internal parameter tuning is not conducted. Here, the internal parameters Γ_σ and Γ_γ and the initial γ and σ values were tuned on the basis of a prior study from the authors.[77]

A parametric study of the input space in terms of the embedding dimension and time delay was conducted on both the high- and low-rate data. The parametric study was conducted by running the estimator over a grid of possible input space $d = [2{:}1{:}10]$ and $\tau = [1{:}1{:}400]$ to find the optimal values for d and τ (Equation 1 that minimized J_1). We find $d = 3$ and $\tau = 46$ for the high-rate data and $d = 2$ and $\tau = 150$ for the low-rate data. The fitting errors using these values are plotted in Figure 5. It can be noted that the optimal dimension d decreases with a smoother function (e.g., low-rate data) and that the smoother function leads to a significantly lower estimation error, yielding $J_1 = 0.238$ for the high-rate data and $J_1 = 0.106$ for the low-rate data. Also, the estimator for the filtered data converges 0.105 ms faster. The lower performance of the estimator on the raw data is explained by the larger and higher nonlinearities in the estimated dynamics.

The influence of the input space on the performance of the estimator is further studied by investigating different combinations of d and τ. The combinations and associated performance J_1 and J_2 are listed in Table 2, in which the change in performance or percent difference in J_1 and J_2 relative to the optimal performance is indicated in columns 6 and 8, respectively. Note that a dimension $d = 2$ is considered as the lowest possible dimension, and therefore, the under embedding is not shown for the low-rate data.

Results from Table 2 show that utilizing the wrong input space for the high-rate dynamics results in larger decreases in performance J_1 when compared with the utilization of the wrong input space for the low-rate dynamics. This is similar for the J_2 performance metric, except for the effect of over embedding, which is important for both the high- and low-rate systems yet larger for the low-rate system. For the high-rate dynamics, under embedding yields to a better performance when compared with over embedding. This can be explained by a faster convergence provided by a lower input space dimension, because a function of lower dimension can be populated faster with sequential training examples. Using the wrong τ has dramatic effects on the performance of the high-rate estimator. Figure 6 is a plot of the absolute estimation

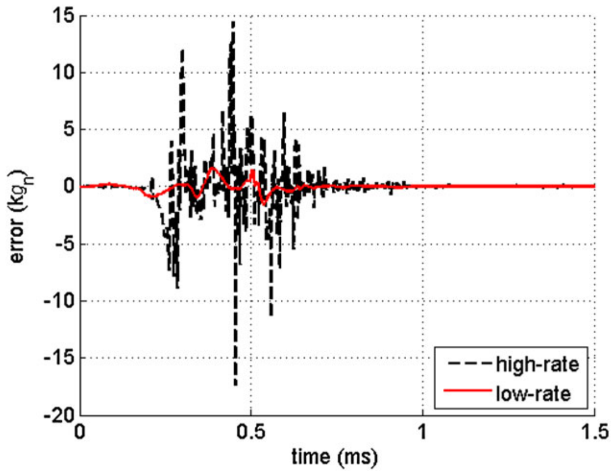

FIGURE 5 Comparison between estimation errors of high- and low-rate data

TABLE 2 Estimator performance associated with different input spaces

Data	Combination	d	τ	J_1	% diff J_1	J_2 (ms)	% diff J_2
High rate	Optimal	3	46	0.238	—	0.484	—
High rate	Under embedding	2	46	0.310	30.3	0.542	12.0
High rate	Over embedding	4	46	0.454	90.8	0.876	81.0
High rate	Incorrect information	3	150	0.441	85.3	0.681	40.7
Low rate	Optimal	2	150	0.106	—	0.379	—
Low rate	Over embedding	3	150	0.143	34.9	0.748	97.4
Low rate	Incorrect information	2	46	0.141	33.0	0.446	17.7

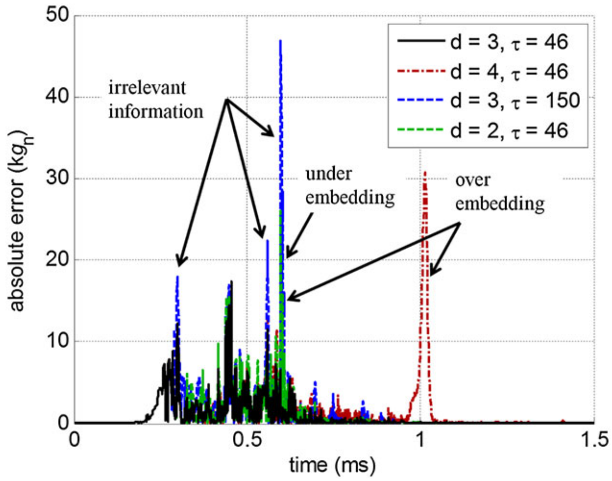

FIGURE 6 Absolute estimation errors for different input strategies for high-rate dynamics

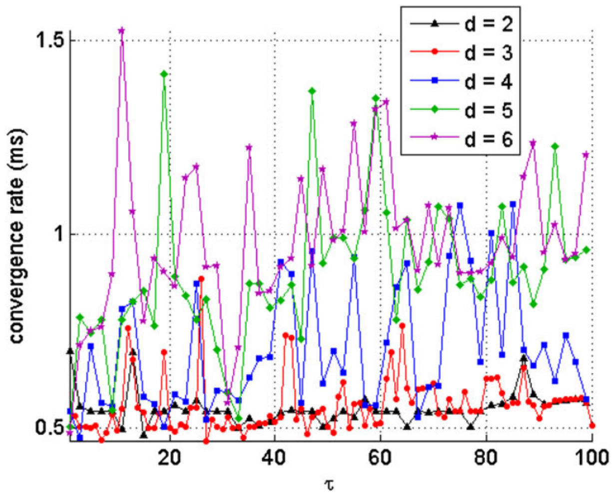

FIGURE 7 Convergence rates as a function of embedding dimensions and time delays for high-rate dynamics

errors of the high-rate dynamics under different input spaces. The absolute estimation error is used to portray the errors associated with incorrect input spaces to make the plots more readable. An input space of incorrect time delay ($d = 3$, $\tau = 150$) yields significantly higher error peaks compared with other strategies. Under embedding ($d = 2$, $\tau = 46$) produces a large error peak at about 0.6 ms, the same as for the incorrect time delay but around 50% lower in magnitude. Over embedding ($d = 4$, $\tau = 46$) also results in high peaks, yet of relatively smaller magnitude. The error peak at 0.6 ms is about equal to the under embedding error, and the other peak occurs after the convergence with the other strategies. The presence of these abnormal high error peaks can lead to false alarms and incorrect decisions in the closed-loop process.

Figure 7 shows how the convergence rate is affected by the choice of different d and τ. The results show that different input spaces within the provided range can yield a difference in convergence rate over 1 ms, which is very large for high-rate dynamics. Note that a convergence rate beyond 1.5 ms means that the estimator never converged, and the simulation was stopped, because the high-amplitude dynamic response is approximately 1 ms. The optimal convergence rate is given by the choice $d = 3$ and $\tau = 27$ yielding a convergence rate of 0.467 ms (a 3.5% increase in performance), which differs from the input space providing the best 2-norm error (J_1) and yields $J_1 = 0.316$ (a 32.8% decrease in performance). The choice of lower dimensions d generally yield a more constant convergence rate for varying τ.

Simulations of the estimation using the neuro-observer were also conducted on experimental data from a second impact event. The second impact event was created with a drop height of 72 in. and 1/2 in thickness felt, instead of a drop height of 20 in. and 1/16 in thickness felt for the first impact. In reality, different environmental conditions could occur even from a slight change in impact location or angle, for example, that can produce very different results by exciting different

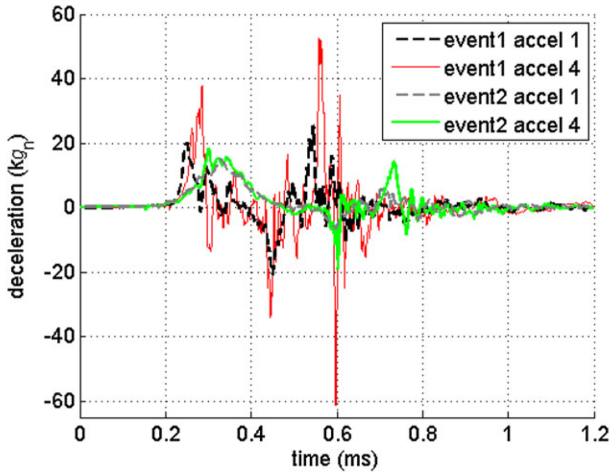

FIGURE 8 Dynamic Events 1 and 2

TABLE 3 Optimal d and τ for Events 1 and 2

Dynamic event	combination	d	τ	J_1	% diff J_1	J_2 (ms)	% diff J_2
Event 1	Optimal	3	46	0.238	—	0.484	—
Event 1	Incorrect	4	54	0.435	82.8	1.027	112.2
Event 2	Optimal	4	54	0.196	—	0.490	—
Event 2	Incorrect	3	46	0.256	30.6	0.637	30.0

modes with different phases. The optimal input space was determine for both events, independently. Figure 8 shows the different dynamic events.

The optimal d and τ for Events 1 and 2 are listed in Table 3.

The performance attained using optimal input space, optimized for the J_1 metric, for Events 1 and 2 was compared. Results are listed in Table 3. Using the optimal input space of Event 1 for Event 2, there is a 30% decrease in performance for both J_1 and J_2. A significantly worst result is observed seen when using the optimal input for Event 2 for Event 1. These results show that although a static input space could be designed based on an event, it might be very ineffective when used for state estimation of the same system subjected to a different event.

5 | CONCLUSION

State estimation of high-rate dynamics is a challenging task, in particular for complex engineering systems experiencing highly dynamic events and requiring real-time observability to ensure adequate performance. Application of the estimators on the high-rate systems include hypersonic vehicles and impact protection systems. High-rate system state estimation-specific challenges were listed, and the applicability of typical observers at achieving that specialized task were discussed. A path to state estimation of high-rate dynamics was presented. It was argued that data-based AOs could be particularly promising, because of their ability to process information without knowledge of system dynamics, or the high-rate event, and their high adaptability to complex dynamics. However, the utilization of AOs comes with the cost of slower convergence rates, a critical obstacle to the state estimation of high-rate dynamics problem.

A potential solution to improving the convergence rate of AOs is through the careful design of the input space of the estimator. The influence of the input space on a high-rate state estimator was investigated. An adaptive neuro-observer was constructed, and simulations were performed on high-rate laboratory experimental data. The estimator's performance was based on the normalized 2-norm errors and convergence rates. Simulation results highlighted a few key observations:

- The use of a proper input space is critical to enhancing the performance of high-rate dynamics state estimators. The use of incorrect input space for estimation has significant negative impacts, which are much larger for high-rate data than low-rate data.

- The misrepresentation of the system dynamics through wrong input spaces produces abnormally large error peaks, which can lead to poor decisions in the closed-loop process.
- The input space is not unique to a system but rather to a dynamic environment. Different input spaces are required for different events for optimal estimator performance.

From the simulation results, the importance of input space is evident in making the most accurate and fast estimations. The challenge is choosing the right input space for the right situation. Often, the type of high-rate dynamic event a system will experience before the event occurs is unknown, and pre-selecting the input space is a very challenging task. A solution, part of future work, is to develop algorithms automating the input space selection process as a function of different events, yielding adaptive input spaces.

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